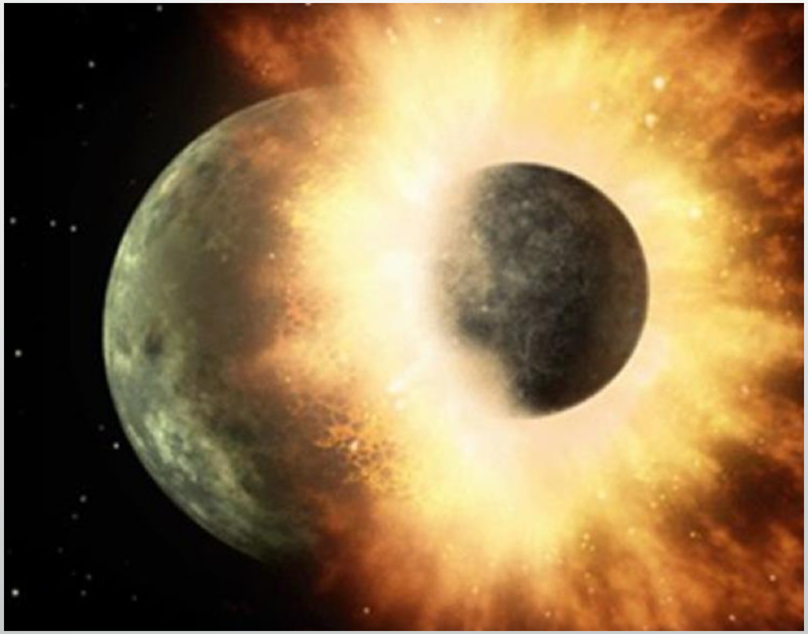
Forces and Momentum



Year 12 Physics

Force

- A force is a push or a pull (year 9 definition)
- It is measured in Newtons
- It is measured by its effect

$$\vec{F} = m\vec{a}$$

- Which of course is Newton’s second law
- Note, things don’t have to be connected to impose forces;
  - Gravity
  - Electrostatic forces
  - Nuclear forces

Momentum

Momentum is a property of moving objects and is defined as the product of the mass and the velocity of the object.

Momentum is conserved in an isolated system and may be transferred from one object to other objects when a force acts over a time interval.

$$\vec{\rho} = m\vec{v}$$

Derivation time

Sample question; Starting with Newton's second law, show impulse is equal to the change in momentum

$\vec{F} = m\vec{a}$ *and Impulse* $= \vec{F}\Delta t$

And by definition $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

Therefore $\vec{F} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t}$

and since $\vec{\rho} = m\vec{v}$ then $\Delta\vec{\rho} = m\Delta\vec{v}$

Therefore $\vec{F} = \frac{\Delta\vec{\rho}}{\Delta t}$

And $\vec{F}\Delta t = \Delta\vec{\rho}$

NOTE

Exam questions should state that

"momentum is conserved"

If not – state in your answer;

" assuming momentum is conserved"

Remember: physics calculations are usually models based on assumptions

Or to quote an old B grade movie "*you can't handle the truth*"

$Impulse = \Delta Momentum$

$$Impulse = \vec{\boldsymbol{F}}\Delta \boldsymbol{t}$$

So a 3N force on a brick for 2 seconds =3x2 =6Ns

And also by N2L (an alternative to the last derivation)

$$\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}} \;\; \boldsymbol{where} \;\; \vec{\boldsymbol{a}} = \frac{\boldsymbol{v} - \boldsymbol{v_0}}{\Delta \boldsymbol{t}}$$

$$\therefore \vec{\boldsymbol{F}} = \boldsymbol{m}\frac{\boldsymbol{v} - \boldsymbol{v_0}}{\Delta \boldsymbol{t}}$$

$$and \;\; \vec{\boldsymbol{F}}\Delta \boldsymbol{t} = \boldsymbol{m}\frac{\boldsymbol{v} - \boldsymbol{v_0}}{\Delta \boldsymbol{t}}\Delta \boldsymbol{t} \; (multiply \; both \; sides \; by \; \Delta t)$$

$$\therefore \vec{\boldsymbol{F}}\Delta \boldsymbol{t} = \boldsymbol{m}(\boldsymbol{v} - \boldsymbol{v_0})$$

$$\therefore \vec{\boldsymbol{F}}\Delta \boldsymbol{t} = \boldsymbol{mv} - \boldsymbol{mv_0}$$

$$\therefore \vec{\boldsymbol{F}}\Delta \boldsymbol{t} = \vec{\boldsymbol{p}} - \vec{\boldsymbol{p}}_{\boldsymbol{0}}$$

Inertia

- Momentum is inertia in motion

Remembering N1L

- Inertia is a measure of how much force it takes to change a body’s motion

Impulse - Momentum

$$Since$$

$$\vec{F}\Delta t = m\Delta\vec{v}$$

$$Therefore$$

$$\vec{F} = \frac{m\Delta\vec{v}}{t}$$

$$Force\ is\ inversley\ propoprtional\ to\ time$$

$$F \propto \frac{1}{t}$$

$$The\ longer\ a\ collison\ takes, the\ lower\ the\ force$$

Thought experiment

- If a very solid 4WD with bulbar hits a big old tree at 100kph
  - The tree doesn't move
  - A 'solid' 4WD doesn't crumple
  - So, the passengers 'stop' almost instantly
- If a standard sedan hits the same tree at 100kph
  - The tree still doesn't move
  - But the car 'crumples'
  - As a result, the passengers "stop slowly"

$$Since\ \boldsymbol{F} \propto \frac{\mathbf{1}}{\boldsymbol{t}}$$

$$Due\ to\ the\ increased\ time\ of\ impact,$$
$$the\ sedan\ passengers\ feel\ less\ force$$

Conservation of momentum

- If you work on the basis that **linear** momentum is conserved, you will be okay.
  - Momentum is a linear property
  - It is a vector measurement
  - It should be dealt with in vector components
  - Each vector component will be conserved

Types of collisions

- If two objects collide, there are a number of possible outcomes
  - They may stick together and move as one
  - They may ‘bounce’ off each other
  - Though we often treat them in a simple way, they actually have very different physics.

A stick together collision

- An object going at 6.00ms^{-1} to the right, hits an object going at 8.00ms^{-1} to the left

- For simplicity, lets say they both have a mass of 2.00kg

$$\vec{p}_i = m_r v_r + m_b v_b$$

$$\vec{p}_i = 2.00 \times 6.00 + 2.00 \times (-8.00)$$

$$\vec{p}_i = -4.0Ns$$

and since momentum is conserved

$$\vec{p}_i = \vec{p}_f$$

stick together collision, cont.

$$We\ have\ established$$

$$\vec{p}_f = \vec{p}_i = -4.00Ns$$

$$m_f = m_r + m_b = 4.00kg$$

$$and\ \vec{p}_f = m_f\vec{v}_f$$

$$so\ \vec{v}_f = \frac{\vec{p}_f}{m_f}$$

$$\vec{v}_f = -\frac{4.00}{4.00}$$

$$\vec{v}_f = -1.00ms^{-1}(to\ the\ right)$$

But, lets not stop there …

- What about Kinetic Energy?

$$E_{ki} = E_{kri} + E_{kbi}$$

$$E_{ki} = \frac{1}{2} \times 2.00 \times 6.00^2 + \frac{1}{2} \times 2.00 \times 8.00^2$$

$$E_{ki} = 100.J$$

$$and$$

$$E_{kf} = E_{k(r+b)f}$$

$$E_{ki} = \frac{1}{2} \times 4.00 \times (-1.00)^2$$

$$E_{ki} = 2.00J$$

Energy was not conserved!

- Aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaah!
- Firstly, we call this an “**inelastic collision**”
  - Momentum is conserved
  - Kinetic Energy is not conserved
- Think about all you know about entropy
  - In any energy transformation or transfer, if energy can go to a less ordered state; it will.
  - The energy is not lost, it is simply no longer kinetic energy.

If energy is conserved …

- Then we call it an “**elastic collision**”

- Lets consider the previous collision, but this time, let the objects bounce off each other
- Case 1 – bouncing back in opposite directions
- Thought experiment;

  Conservation of momentum requires $\vec{p}_f = -4Ns$

  If the masses are both 2.00kg and they are travelling in opposite directions, momentum will be conserved if the velocities are anything as long as one is travelling to the right at 2.00ms-1 less than the one travelling to the left. (try it for yourself)

Case 1 – bouncing back in opposite directions

But … lets consider the Ek of 3 of those cases

We already know $E_{ki} = 100.J$

Case 1; Red bounces back at 6.00ms^{-1}, blue at 4.00ms^{-1}

$$E_{ki} = \frac{1}{2} \times 2.00 \times 6.00^2 + \frac{1}{2} \times 2.00 \times 4.00^2 = 52.0J$$

Case 2; Red bounces back at 8.00ms^{-1}, blue at 6.00ms^{-1}

$$E_{ki} = \frac{1}{2} \times 2.00 \times 8.00^2 + \frac{1}{2} \times 2.00 \times 6.00^2 = 100.J$$

Case 3; Red bounces back at 10.00ms^{-1}, blue at 8.00ms^{-1}

$$E_{ki} = \frac{1}{2} \times 2.00 \times 10.00^2 + \frac{1}{2} \times 2.00 \times 8.00^2 = 164\text{J}$$

$$only\ the\ case\ where\ velocities\ are\ 'matched'\ is\ elastic$$

While you are thinking

- Which is most likely to occur in the 'real' world

- Case 1

  Because …

  Entropy

- Case 3 cant happen – where did an increase in energy come from?
- Case 2 could happen, but … "no energy losses in the 'real world'". Really?